

Logic synthesis is the task of designing circuits at the level of gates and wires to meet a specification. As a research area, it is at once mature and wide-open. It is mature in the sense that progress in the area is slow and incremental. It is wide-open in the sense that even the best available tools are based on heuristics; these produce results that everyone admits are probably far from optimal.

In this problem you will explore some of the basic tasks of logic synthesis. Consider the following target functions:

$$\begin{aligned}f_1 &= \bar{x}_1x_2\bar{x}_3 + x_1\bar{x}_2\bar{x}_3 + x_1\bar{x}_2x_3 + \bar{x}_1\bar{x}_2x_3, \\f_2 &= \bar{x}_1\bar{x}_2\bar{x}_3 + x_1x_2x_3 + x_1x_2\bar{x}_3 + x_1\bar{x}_2x_3, \\f_3 &= \bar{x}_1x_2\bar{x}_3 + \bar{x}_1\bar{x}_2\bar{x}_3 + x_1\bar{x}_2\bar{x}_3 + \bar{x}_1\bar{x}_2x_3.\end{aligned}$$

Our cost measure for area will be the number of *literals* in the algebraic expressions – that is to say, the number of appearances of variables, without regard to negations. In the expressions above, each function has a cost of 12, for a total cost of 36.

Two-Level Logic Minimization

The first step in logic design is to simplify the Boolean expressions individually, if possible. In the *sum-of-products* (SOP) form, a Boolean expression is formulated as the OR (disjunction) of AND (conjunctive) terms. Minimal in this context means with the fewest conjunctive terms, and the fewest literals per conjunctive term.

Problem [0.5 points]

Obtain minimal SOP forms for the target functions. (Solution has total area cost of 20.)

Two-Level Logic Minimization with Shared Terms

When minimizing several functions jointly, one can often save in area by sharing terms. For instance, the term

$$c_1 = x_1\bar{x}_2x_3$$

can be used in both the expressions for f_1 and f_2 .

Problem [1.0 points]

Obtain minimal SOP expressions, sharing terms among them. (Solution has total area cost 17.)

Multi-Level Logic

In multi-level designs, an arbitrary structure is permitted. Algebraically, a factored form is a parenthesized expression of OR and AND operations. For instance, the expression

$$x_1 + x_2x_3 + x_2x_4$$

can be written as

$$x_1 + x_2(x_3 + x_4).$$

Problem [1.0 points]

Find minimal factored forms for the target functions. (Solution has total area cost 17.)

Substitution Orderings

Beyond sharing of terms in SOP expressions and factoring, arbitrary substitutions can be made in multi-level logic synthesis. For instance, for the target functions, we can express f_1 in terms of f_2 and f_3 ,

$$f_1 = \bar{x}_2x_3 + \bar{f}_2f_3.$$

Problem [1.5 points]

Exploit such substitutions to obtain the least costly expressions for the target functions. (Solution has a total area cost of 14.)